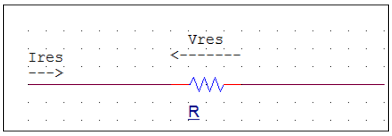
From Schematics to Block diagrams

Introduction

In our examples, we will only use resistors, capacitors, inductors and op-amps. Before we begin we need to remind ourselves of the characteristics of each of those components.

1. Passive component characteristics

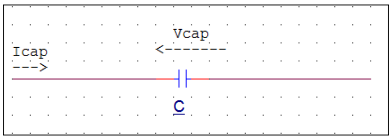
   (a) Resistor

   Time domain equation: $V_{RES}(t) = R \cdot I_{RES}(t)$

   Laplace domain equation: $L\{V_{RES}(t)\} = L\{R \cdot I_{RES}(t)\} \Rightarrow$
   $V_{RES}(s) = R \cdot I_{RES}(s)$

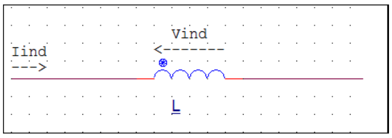
   (b) Capacitor

   Time domain equation: $V_{CAP}(t) = \frac{1}{C} \cdot \int I_{CAP}(t)$

   Laplace domain equation: $L\{V_{CAP}(t)\} = L\{\frac{1}{C} \cdot \int I_{CAP}(t)\} \Rightarrow$
   $V_{CAP}(s) = \frac{1}{C \cdot s} \cdot I_{CAP}(s)$

   (c) Inductor

   Time domain equation: $V_{IND}(t) = L \cdot \frac{d}{dt}(I_{IND}(t))$

   Laplace domain equation: $L\{V_{IND}(t)\} = L\{L \cdot \frac{d}{dt}(I_{IND}(t))\} \Rightarrow$
   $V_{IND}(s) = L \cdot s \cdot I_{IND}(s)$

2. Ideal Op-amp

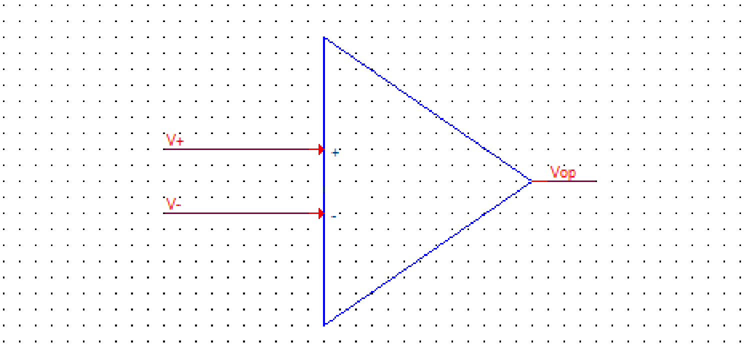


Figure 1: **Ideal Op-amp Schematic**

The operation of ideal op-amp is characterized by the equation:

(a) $V_{OP} = A \cdot (V_{+} - V_{-})$, where A is called the open-loop gain of the op-amp.

In the ideal case, we assume that $A = K$, where K is a very large constant.

The operation of an ideal op-amp can, therefore, be represented by the following block diagram:

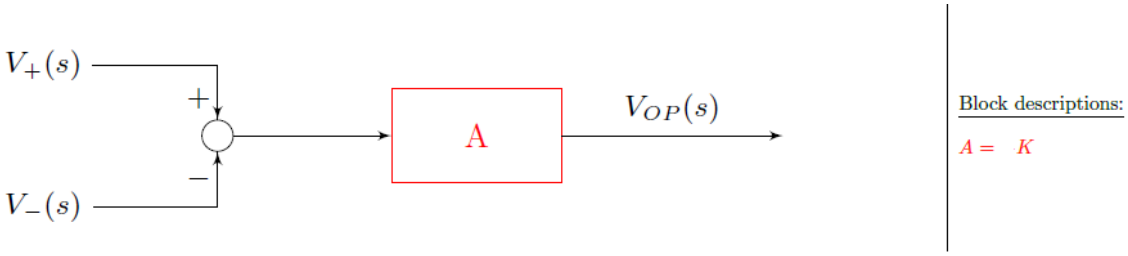


Figure 2: **Ideal Op-amp Block Diagram**

Examples using Ideal Op-Amps

1. Voltage Follower (Ideal Gain: $G = 1$)

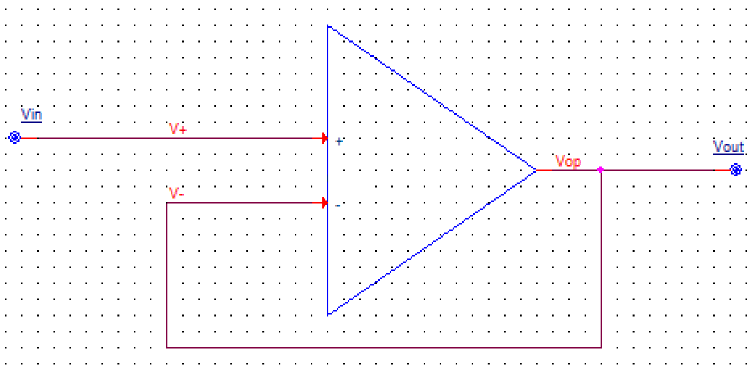


Figure 3: **Voltage Follower Schematic**

To convert this into a block diagram we will utilize the op-amp block diagram derived previously. All we need is to find V_+, V_- and V_{OP} in terms of V_{IN} and V_{OUT} and remember that, by definition, $V_{OP} = A \cdot (V_+ - V_-)$. Let us demonstrate:

(a) $V_+ = V_{IN}$, we can rewrite V_+ as V_{IN} in in our block diagram.

(b) $V_- = V_{OUT}$, we can connect V_{OUT} to V_- directly in our block diagram (Feedback Loop!)

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = K$

(d) $V_{OUT} = V_{OP}$, we can rewrite V_{OP} as V_{OUT} in our block diagram.

We can combine the above equations to draw the voltage follower in block diagram format:

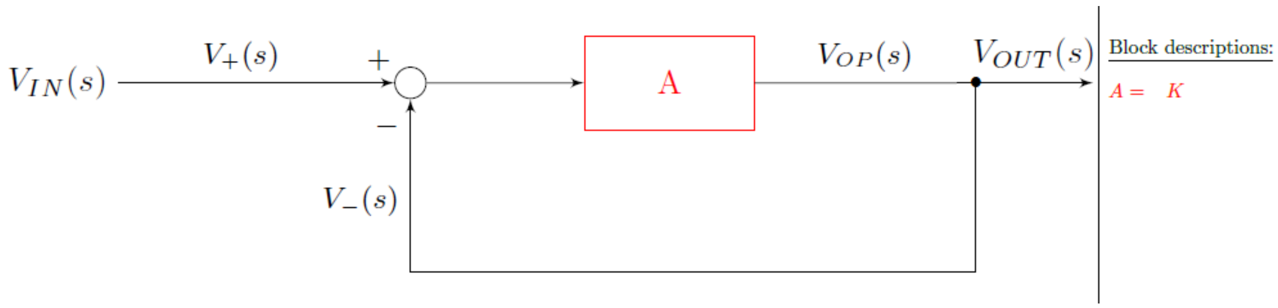


Figure 4: **Voltage Follower Block Diagram**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{A}{1+A}$

From that we can determine the Ideal Gain:

$G = \lim_{K \to \infty} H(s) = 1$

2. Non-Inverting Amplifier (Ideal Gain: $G = 1 + \frac{R_2}{R_1}$)

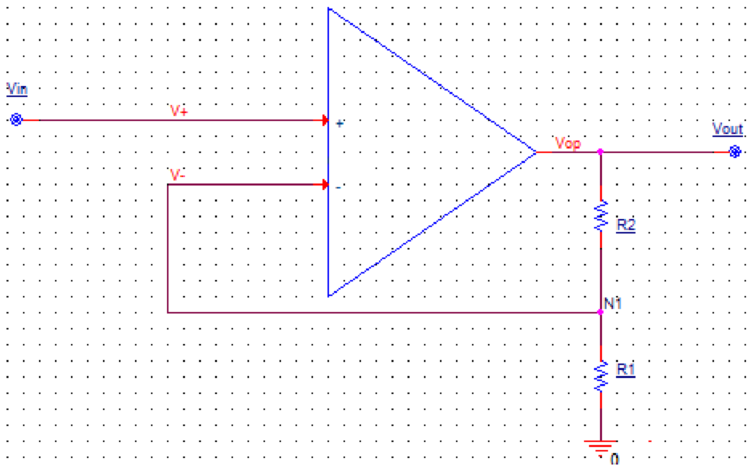


Figure 5: **Non-Inverting Amplifier Schematic**

Let us now, once again, try to extract the necessary equations to construct our block diagram.

(a) $V_+ = V_{IN}$, we can rewrite V_+ as V_{IN} in in our block diagram.

(b) KCL @ N1: $\frac{V_-}{R_1} = \frac{V_{OUT}}{R_1+R_2} \Rightarrow V_- = \frac{R_1}{R_1+R_2} \cdot V_{OUT}$

We can now define $\beta = \frac{R_1}{R_1+R_2}$ and place it between V_{OUT} and V_- in our block diagram.

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = K$

(d) $V_{OUT} = V_{OP}$, we can rewrite V_{OP} as V_{OUT} in our block diagram.

We observe that the only difference to the voltage follower is the block β in the feedback loop:

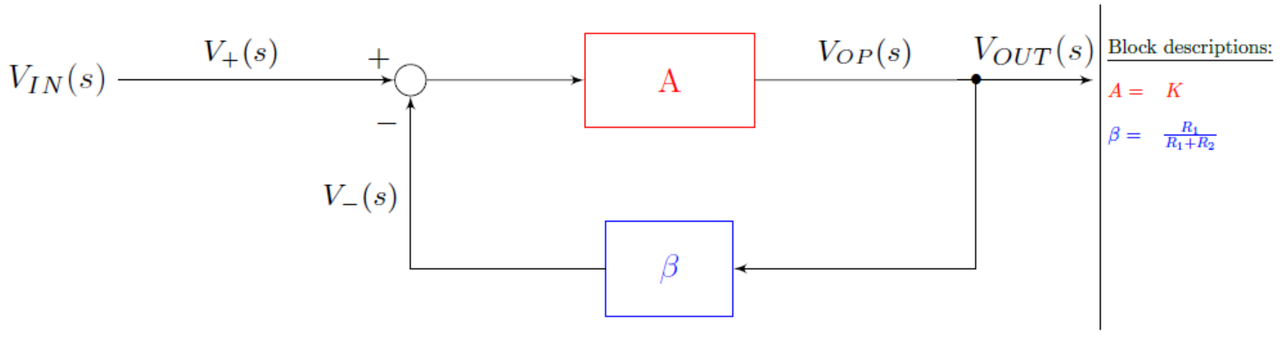


Figure 6: **Non-Inverting Amplifier Block Diagram**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{A}{1+\beta \cdot A}$

From that we can determine the Ideal Gain:

$G = \lim_{K \to \infty} H(s) = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$

TASK 1

(a) Given that $G = \frac{1}{\beta}$, rewrite $H(s)$ as a function only of G and K.

(b) For $K = 10^5$, use MATLAB to sketch the bode plot of $H(s)$ for $G = 1, 10^2, 10^4$.

(c) Define the error $e = |\frac{G-H(s)}{G}|$. Determine the maximum value of G, such that $e < 1\%$.

3. Inverting Amplifier (Ideal Gain: $G = -\frac{R_2}{R_1}$)

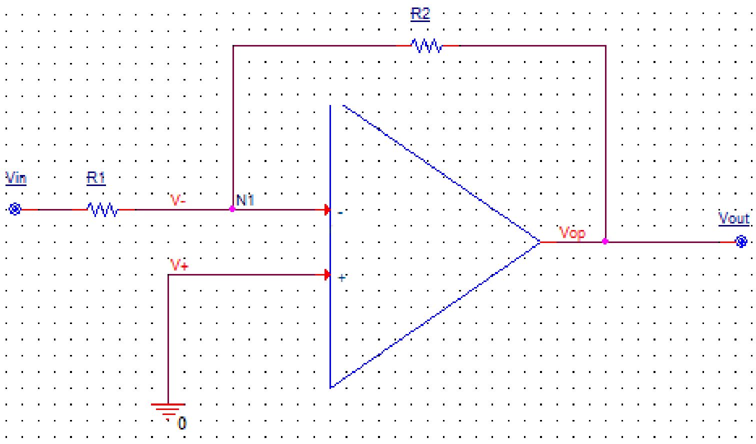


Figure 7: **Inverting Amplifier Schematic**

Relevant equations:

(a) $V_+ = 0$.

(b) KCL @ N1: $\frac{V_- - V_{IN}}{R_1} = \frac{V_{OUT} - V_-}{R_2} \Rightarrow V_- = \{\frac{R_1}{R_1+R_2}\} \cdot V_{OUT} - \{-\frac{R_2}{R_1+R_2}\} \cdot V_{IN}$

We can see that $\beta = \frac{R_1}{R_1+R_2}$ defines again the feedback relation between V_- and V_{OUT}.

We need to define $\alpha = -\frac{R_2}{R_1+R_2}$ (it describes the relation between V_- and V_{IN}).

The fact that $V_- = \beta \cdot V_{OUT} - \alpha \cdot V_{IN}$ will be represented by a node.

Finally, defining α as a negative quantity is not necessary, but it is a trick that allows minimum differences between the block diagram of the inverting and non-inverting amplifier.

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = K$

(d) $V_{OUT} = V_{OP}$, we can rewrite V_{OP} as V_{OUT} in our block diagram.

We can now connect all the blocks together to create a preliminary block diagram.

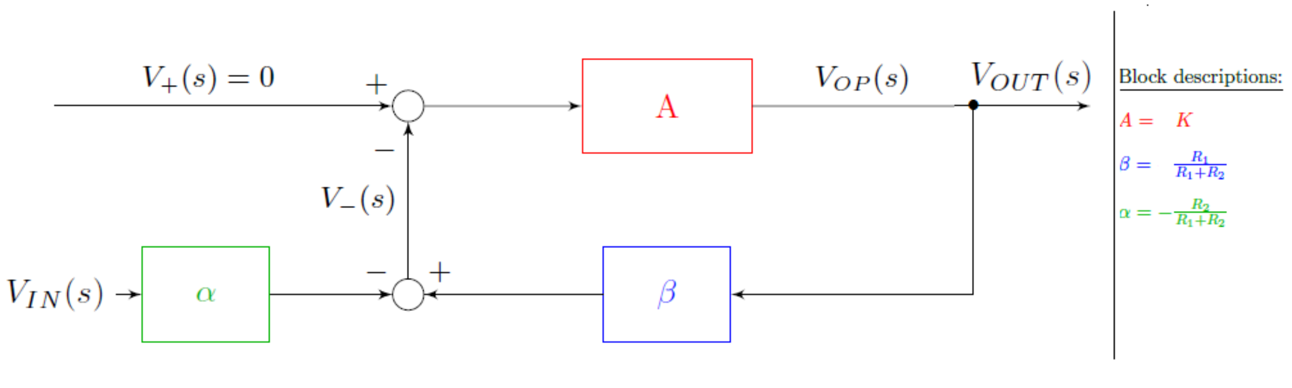


Figure 8: **Inverting Amplifier Block Diagram (Preliminary)**

We can further simplify our block diagram into the following.

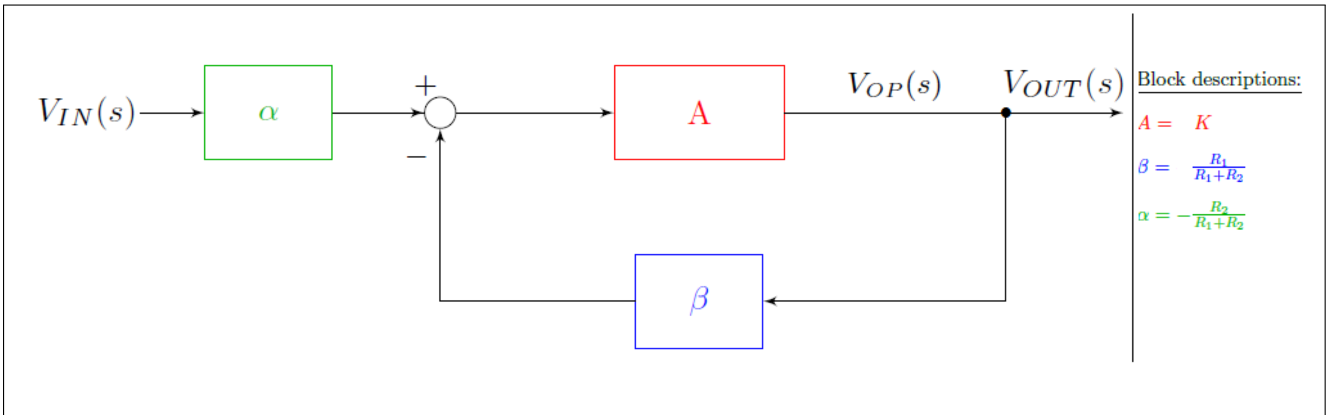


Figure 9: **Inverting Amplifier Block Diagram**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{\alpha \cdot A}{1+\beta \cdot A}$

From that we can determine the Ideal Gain:

$G = \lim\limits_{K \to \infty} H(s) = \frac{\alpha}{\beta} = -\frac{R_2}{R_1}$

<u>TASK 2</u>

(a) Given that $|G| = \frac{R_2}{R_1}$, rewrite both α and β as a functions only of $|G|$.

(b) Use your answers to part (a) to rewrite H(s) as a function only of $|G|$ and K.

(c) For $K = 10^5$, use MATLAB to sketch the bode plot of $H(s)$ for $G = 1, 10^2, 10^4$.

A more realistic Op-Amp model

Up to this point in our analysis, we have been treating op-amps as components of:

(a) infinite input impedance (no current goes in to the + or - terminals)

(b) frequency independent open-loop gain $A = K$.

As well as completely ignoring:

(c) the op-amp output impedance (and its effects depending on the load being driven).

(d) numerous other shortcomings of real op-amps (such as bias currents, noise etc.).

As this is not an EE module, we will provide an op-amp model that only accounts for (b) and (c). **While this model is by no means complete**, it is much closer to reality than the ideal case and provides for interesting examples.

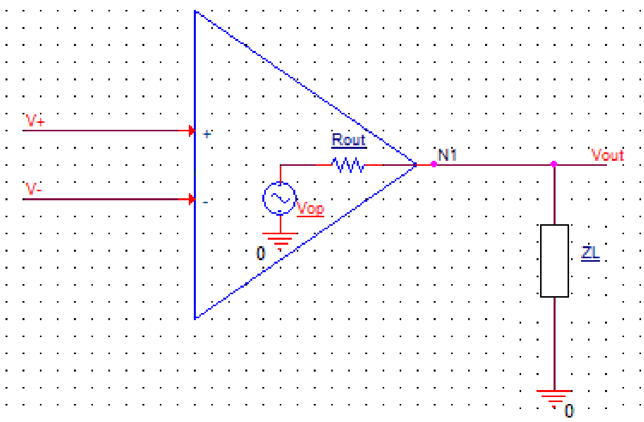


Figure 10: **Non-Ideal Op-Amp Schematic**

To avoid additional complexity we will use the dominant pole approximation of the (now frequency dependant) open-loop gain: $A = \frac{K}{\tau \cdot s+1}$, where K is still a very large constant, but now there is a pole at $s = -\frac{1}{\tau}$.

(a) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = \frac{K}{\tau \cdot s+1}$

As the output impedance is no longer zero, we need to define a load of arbitrary impedance Z_L and include its effects in our calculations.

(Note that the load is not necessarily a resistor. It could be any combination of passive or active components).

(b) KCL @ N1: $\frac{V_{OP}-V_{OUT}}{R_{OUT}} = \frac{V_{OUT}}{Z_L} \Rightarrow V_{OUT} = \frac{1}{1+\frac{R_{OUT}}{Z_L}} \cdot V_{OP}$

We can now define $\gamma = \frac{1}{1+\frac{R_{OUT}}{Z_L}}$ and place it between V_{OP} and V_{OUT}.

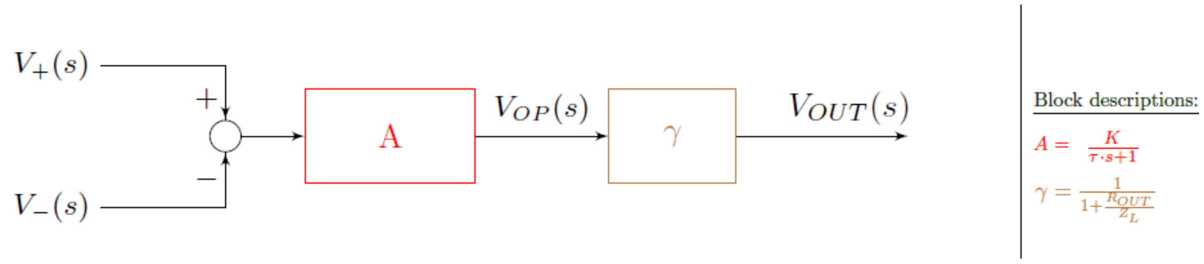


Figure 11: **Non-Ideal Op-Amp Block Diagram**

Then, $H(s) = \frac{V_{OUT}(s)}{V_+(s) - V_-(s)} = A \cdot \gamma$

TASK 3

For $K = 10^5, \tau = 0.1$ and $R_{OUT} = 100\Omega$, use MATLAB to sketch the bode plot of H(s) when:

(a) the load is a resistor $R_L = 1\Omega$, 100Ω, $10\ k\Omega$.

(b) the load is a capacitor $C_L = 100pF$, $10nF$, $1\mu F$.
(remember that the impedance of capacitor C is $\frac{1}{C \cdot s}$)

Examples using a more realistic Op-Amp model

1. Voltage Follower (Ideal Gain: $G = 1$)

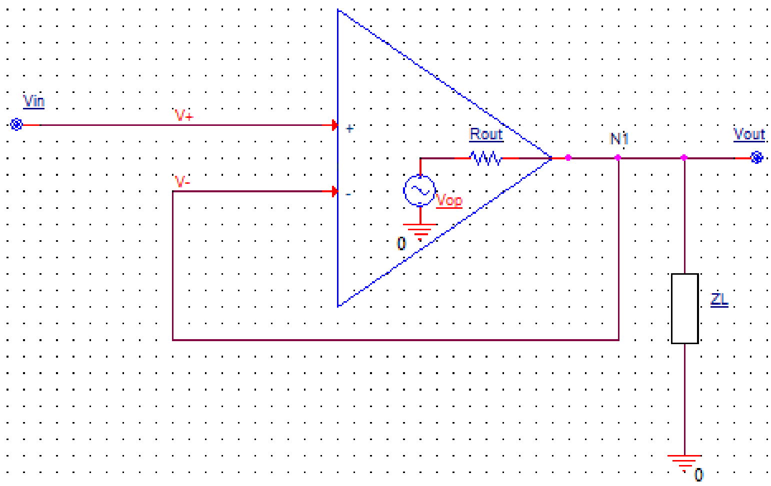


Figure 12: **Voltage Follower Schematic**

Relevant equations:

(a) $V_+ = VIN$.

(b) $V_- = VOUT$

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = \frac{K}{\tau \cdot s+1}$

(d) KCL @ N1: $\frac{V_{OP}-V_{OUT}}{R_{OUT}} = \frac{V_{OUT}}{Z_L} \Rightarrow V_{OUT} = \frac{1}{1+\frac{R_{OUT}}{Z_L}} \cdot V_{OP}$

We can see that $\gamma = \frac{1}{1+\frac{R_{OUT}}{Z_L}}$ defines again the relation between V_{OP} and V_{OUT}.

Corresponding Block Diagram:

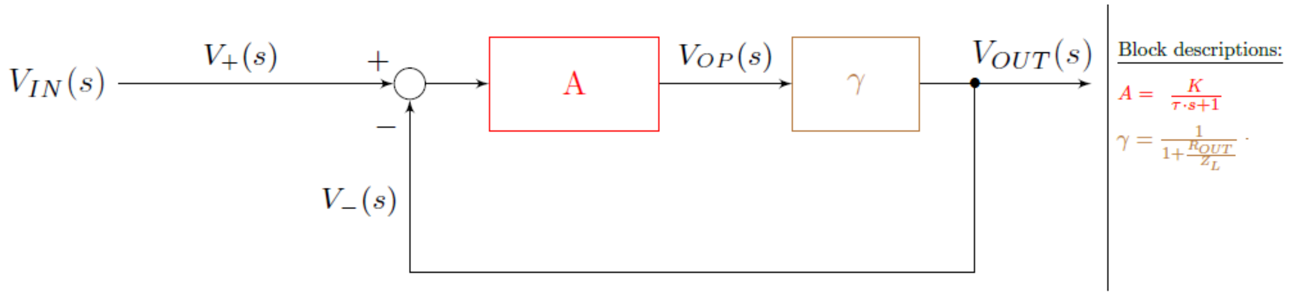


Figure 13: **Non-Ideal Voltage Follower Block Diagram**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{A \cdot \gamma}{1+A \cdot \gamma}$

From that we can determine the Ideal Gain:

$G = \lim_{K \to \infty} H(s) = 1$

TASK 4

(a) By examining the open-loop bode plots from the previous task, determine whether the voltage follower is in danger of instability when the load is:

i. a resistor ($Z_L = R_L$).
ii. a capacitor ($Z_L = \frac{1}{C_L \cdot s}$).

To accomplish this observe the gain and phase margin in each case.

Bonus question: How many poles does the open loop transfer function have in each case? How is this relevant in our stability analysis?

(b) For $K = 10^5, \tau = 0.1$ and $R_{OUT} = 100\Omega$, use MATLAB to sketch the bode plot of the transfer function H(s) when the load is:

i. is a resistor $R_L = 1\Omega$, 100Ω, $10\ k\Omega$

ii. is a capacitor $C_L = 100pF$, $10nF$, $1\mu F$

Does this confirm your findings from part (a)?

2. Non-Inverting Amplifier (Ideal Gain: $G = 1 + \frac{R_2}{R_1}$)

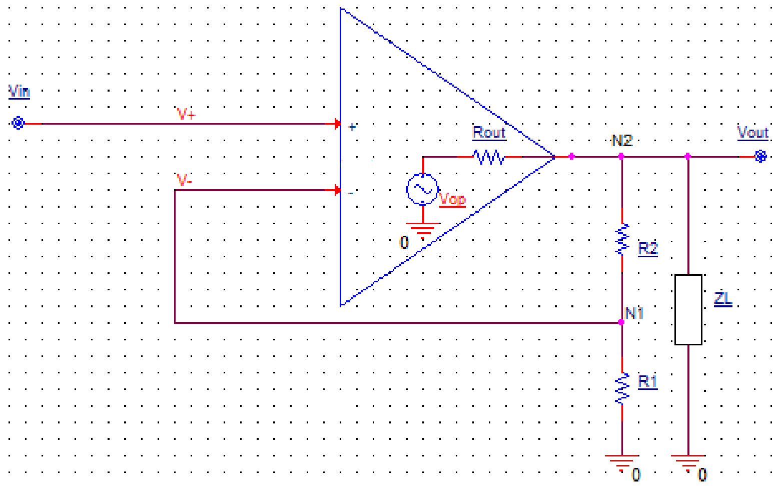


Figure 14: **Non-Inverting Amplifier Schematic**

Relevant equations:

(a) $V_+ = V_{IN}$

(b) KCL @ N1: $\frac{V_-}{R_1} = \frac{V_{OUT}}{R_1+R_2} \Rightarrow V_- = \frac{R_1}{R_1+R_2} \cdot V_{OUT}$

$\beta = \frac{R_1}{R_1+R_2}$ will be placed in the feedback loop.

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = \frac{K}{\tau \cdot s+1}$

(d) KCL @ N2: $\frac{V_{OP}-V_{OUT}}{R_{OUT}} = \frac{V_{OUT}}{Z_L} + \frac{V_{OUT}}{R_1+R_2} \Rightarrow V_{OUT} = \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}} \cdot V_{OP}$

We can now define $\gamma' = \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}}$ and place it in our diagram.

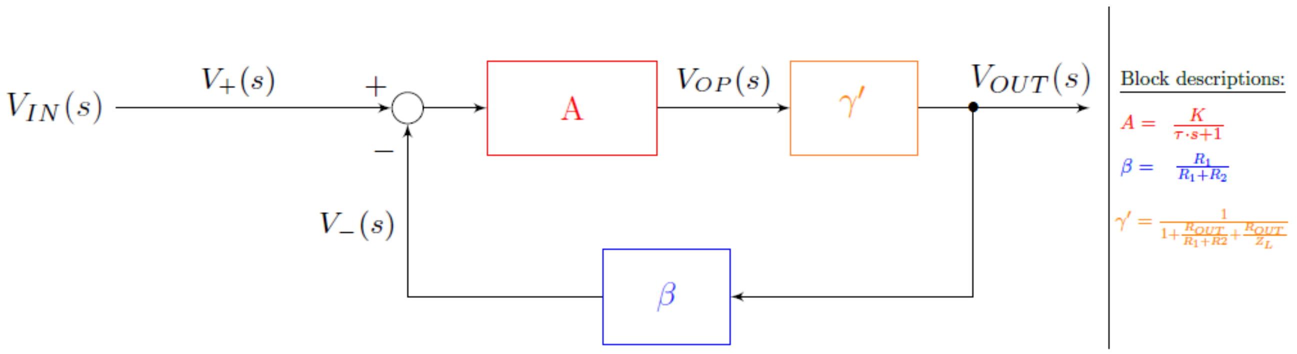


Figure 15: **Non-Inverting Amplifier Block Diagram**

At this point, we can make the very reasonable assumption that $R_1 + R_2 >> R_{OUT}$.

This is possible because the gain of the amplifier is related to the ratio of the two resistors, not their value. As such, any reasonable design will have at least one of R_1 or R_2 be much larger than R_{OUT}.

With that assumption, $\gamma' \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}}$ simplifies to the previously used $\gamma = \frac{1}{1+\frac{R_{OUT}}{Z_L}}$ and our block diagram becomes:

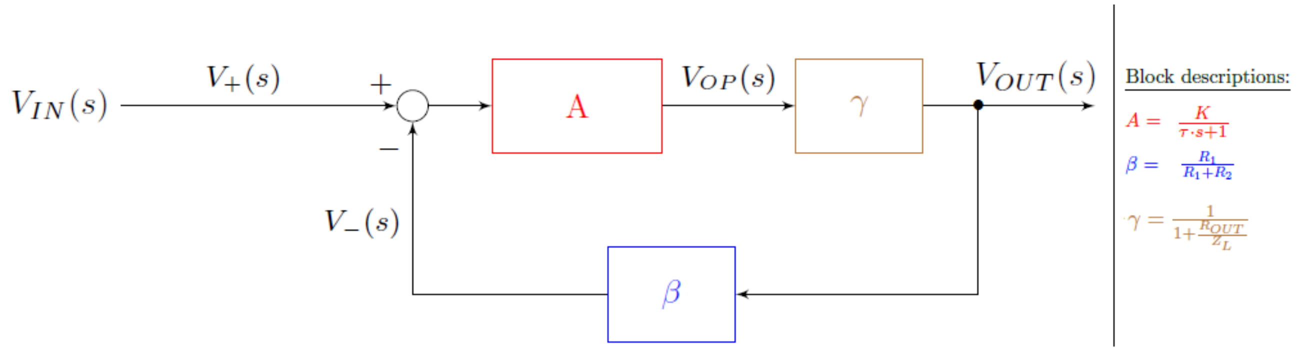


Figure 16: **Non-Inverting Amplifier Block Diagram (simplified)**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{A \cdot \gamma}{1+\beta \cdot A \cdot \gamma}$

From that we can determine the Ideal Gain:

$G = \lim_{K \to \infty} H(s) = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$

<u>TASK 5</u>

For $K = 10^5, \tau = 0.1$ and $R_{OUT} = 100\Omega$:

(a) use MATLAB to sketch the bode plot of $\beta \cdot A \cdot \gamma$ for $G = \frac{1}{\beta} = 1, 10^2, 10^4$ when the load is:

i. a resistor $R_L = 10\ k\Omega$

ii. a capacitor $C_L = 1\mu F$

What are the gain and phase margins in each case? Do they change as G increases?

(b) use MATLAB to sketch the bode plot of $H(s)$ for $G = \frac{1}{\beta} = 1, 10^2, 10^4$ when the load is:

i. a resistor $R_L = 10\ k\Omega$

ii. a capacitor $C_L = 1\mu F$

How does the bandwidth vary as G increases?

3. Inverting Amplifier (Ideal Gain: $G = -\frac{R_2}{R_1}$)

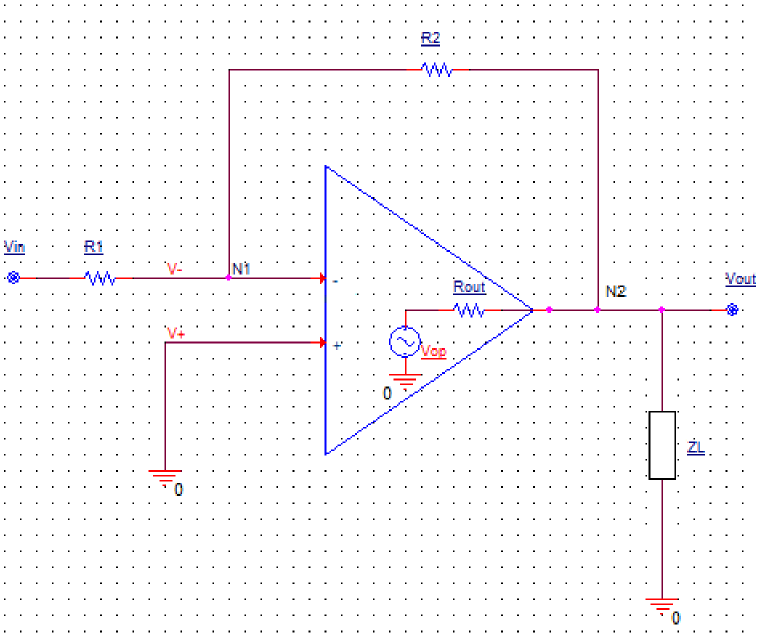


Figure 17: **Inverting Amplifier Schematic**

Relevant equations:

(a) $V_+ = 0$

(b) KCL @ N1: $\frac{V_- VIN}{R_1} = \frac{V_{OUT} - V_-}{R_2} \Rightarrow V_- = \{\frac{R_1}{R_1+R_2} \cdot V_{OUT}\} - \{-\frac{R_2}{R_1+R_2} \cdot V_{IN}\}$

$\beta = \frac{R_1}{R_1+R_2}$ will be placed in the feedback loop.

$\alpha = -\frac{R_2}{R_1+R_2}$ defines the relationship between V_- and V_{IN}.

(c) $V_{OP} = A \cdot (V_+ - V_-)$, where $A = \frac{K}{\tau \cdot s+1}$

(d) KCL @ N2: $\frac{V_{OP} - V_{OUT}}{R_{OUT}} = \frac{V_{OUT}}{Z_L} + \frac{V_{OUT} - V_{IN}}{R_1+R_2} \Rightarrow$

$V_{OUT} = \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}} \cdot V_{OP} + \frac{1}{1+\frac{R_1+R2}{R_{OUT}}+\frac{R_1+R_2}{Z_L}} \cdot V_{IN}$

$\gamma' = \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}}$ describes the relationship between V_{OUT} and V_{OP}.

We must also define $\delta = \frac{1}{1+\frac{R_1+R2}{R_{OUT}}+\frac{R_1+R_2}{Z_L}}$.

This block will describe how V_{OUT} is directly affected by V_{IN} (without passing through all the other stages in our block diagram).

We can now connect all the blocks together to form a preliminary block diagram.

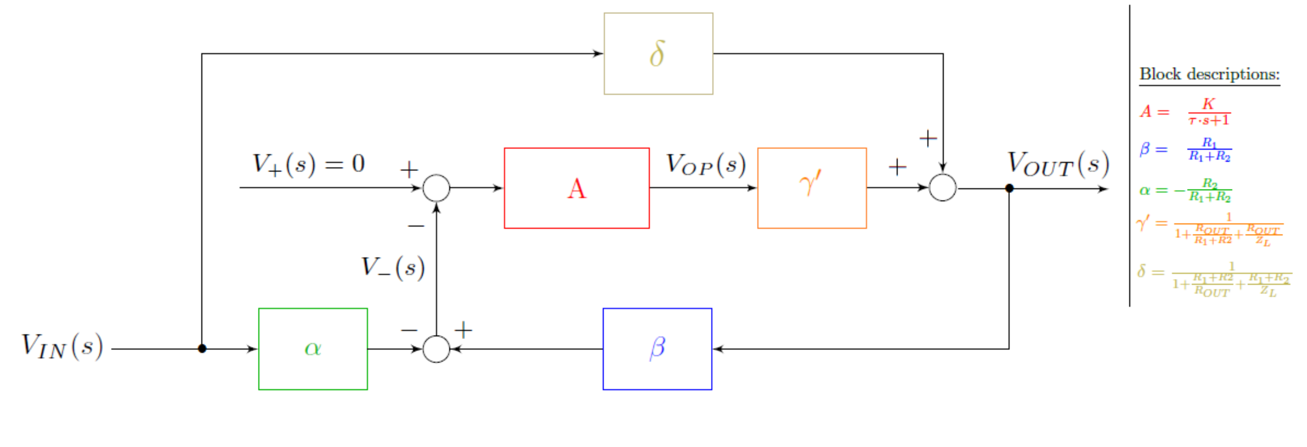


Figure 18: **Inverting Amplifier Block Diagram (preliminary)**

We can further simplify our block into the following.

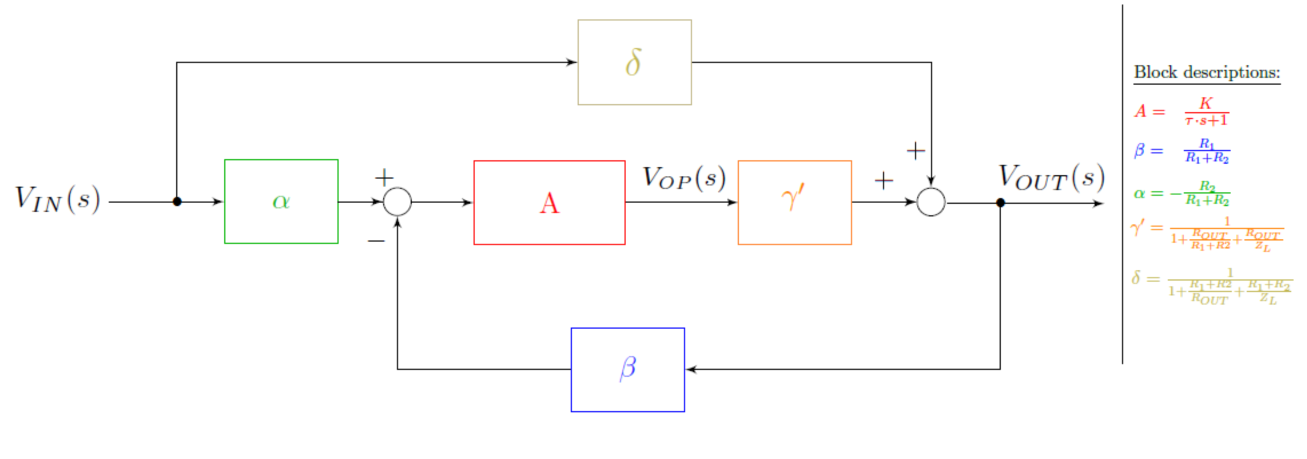


Figure 19: **Inverting Amplifier Block Diagram**

At this point, we can once again make the assumption that $R_1 + R_2 >> R_{OUT}$.

With that assumption, $\gamma' \frac{1}{1+\frac{R_{OUT}}{R_1+R2}+\frac{R_{OUT}}{Z_L}}$ simplifies to the previously used $\gamma = \frac{1}{1+\frac{R_{OUT}}{Z_L}}$ and $\delta = \frac{1}{1+\frac{R_1+R2}{R_{OUT}}+\frac{R_1+R_2}{Z_L}}$ simplifies to 0.

This leads us to the simplified block diagram.

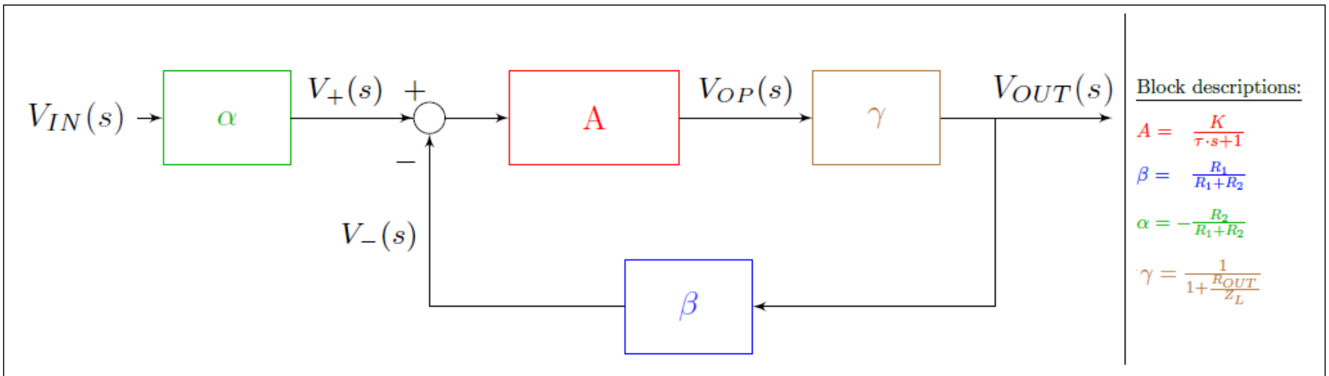


Figure 20: **Inverting Amplifier Block Diagram (simplified)**

By analysing the block diagram we obtain the Transfer function:

$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{\alpha A \cdot \gamma}{1+\beta A \cdot \gamma}$

From that we can determine the Ideal Gain:

$G = \lim_{K \to \infty} H(s) = \frac{\alpha}{\beta} = -\frac{R_2}{R_1}$

TASK 6

To mitigate the stability issues present when driving capacitive loads, it is common practice to place a resistor between the op-amp output and the load. This can be observed in the voltage follower example depicted below:

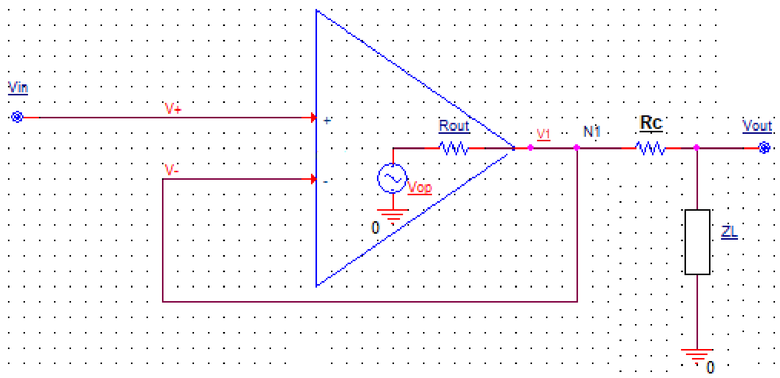


Figure 21: **Compensated Voltage Follower Schematic**

The corresponding block diagram is of the form:

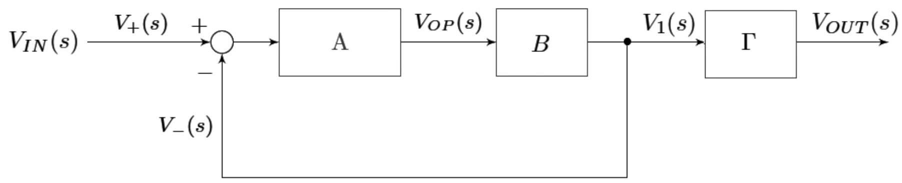


Figure 22: **Compensated Voltage Follower Block Diagram**

Given that the gain of the op-amp is given by: $A = \frac{V_{OP}(s)}{V_{+}(s)-V_{-}(s)} = \frac{K}{\tau \cdot s+1}$

(a) Calculate $H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ in terms of A, B, Γ.

(b) By examining the schematic, determine the equations that describe blocks B and Γ.

(c) For a purely capacitive load ($Z_L = \frac{1}{C_L \cdot s}$):

i. Sketch an estimate of the gain and phase plot of B.

ii. Write down the equation for the phase $\angle B(j\omega)$.

iii. Assuming the circuit designer chose $R_C = 0.5 \cdot R_{OUT}$:

a. Show that the minimum value of the phase occurs at $\omega = \frac{2}{\sqrt{3} \cdot C_L \cdot R_{OUT}}$. Then calculate the minimum value of the phase. (Remember: $\frac{d}{dx}(arctan(\alpha x)) = \frac{\alpha}{\alpha^2 \cdot x^2+1}$)

b. can the phase margin of $A \cdot B$ ever be smaller than 60°? (Regardless of the values of K, τ, R_{OUT}, or C_L)

(d) For $K = 10^5, \tau = 0.1$, $R_{OUT} = 100\Omega$ and $Z_L = \frac{1}{C_L \cdot s}$, use MATLAB to sketch the bode plot of $H(s)$ when $C_L = 1\mu F$ and $R_C = 0\Omega$, 10Ω, 100Ω, $1\ k\Omega$

Which one provides the best performance? (Think about bandwidth and stability)